I. Introduction

We plan to measure the ATA feed patterns using "holographic" measurements with the ATA interferometer. Gerry Harp has studied the patterns of many of the ATA antennas using interferometry with the ATA correlator (Harp, 2009) for the measurements. The basic scheme for an interferometer pair is that one tracks a source with one antenna while making a raster scan with the other, all the while recording the cross-correlation. The correlator output provides the complex voltage pattern of the scanned antenna at all of its pointings. The same can be done with the feed by itself. Although the feed has much less gain than an entire ATA antenna, it is the geometric mean of the gains of the two antennas that determines the sensitivity, and the use of a reference that is an ATA antenna provides sufficient sensitivity.

The strategy is to remove the secondary of one of the ATA antennas and then to measure the correlation of the remaining feed with one of the complete ATA antennas to provide the complex feed pattern of the feed. A bright radio source, such as CassA, will be the radio source. Figure 1 shows an ATA antenna both with and without its secondary mirror removed.

A very bright source, such as a GPS satellite, could be used to produce a raster scan with very high S/N. However, there is an advantage in using a natural radio source with the full 100 MHz bandwidth of the ATA correlator, whereas the bandwidth of the GPS signal is only a few MHz. Conventional feed pattern measurements with a narrow band signal are usually plagued with multipath signal interference in the measurement of the sidelobes of the feed. The use of a wideband signal can reduce the effects of the multipath interference. The cross-correlation of a signal with itself (the autocorrelation function) with different relative delays depends on the bandpass of the signal. The relation is just the Fourier transform. The wide band signal has a correlation delay time interval that is approximately the reciprocal of the bandwidth of the signal. For a correlator bandwidth of 100MHz, the correlation interval is about 10 nsec, or, at the speed of light, 3 meters. Signals that are reflected from nearby structures that have an additional path length of more than 3 meters before arriving at the correlator don't correlate well with the direct signal. The signals that can cause the most trouble will be the reflections from the primary mirror which is very close to the feed. Absorber with at least 10 db one way absorption will be placed on the primary mirror. Reflections from other antennas or the ground will be only weakly detected.

II. Observation

Track CassA with one regular antenna and make a rastor scan with the feed antenna while measuring the cross-correlation of the signals from the two antennas. The received voltage at the reference antenna is $V_1(\theta,\phi,t)$ and at the feed is $V_2(\theta,\phi,t)$. $(\Theta,\phi)$ are polar angles measured from the direction of CassA. The angular extent of CassA is small enough that it would not be resolved at any reasonably short spacing of the two antennas. Thus, the time dependence of the CassA signal voltages at the two antennas is the same. The output from the cross-correlation is:
\[
\int_{t_1}^{t_2} V_1(t,0,0) V_2(t,\theta,\phi) \, dt = \langle V_2(\theta,\phi) \rangle, \quad \text{the feed RMS voltage pattern.} \tag{1}
\]

In the theoretical pattern for the feed there are essentially no nulls. The lowest value of the feed electric field is .05 of the peak, and we need enough sensitivity to accommodate this dynamic range of about 26 dB with some margin.

III. Expected sensitivity in the pattern measurements.

a) ATA antenna effective area: The average aperture efficiency is 0.6, and the corresponding effective area is
\[
A_{e1} = 0.6 \left( \frac{\pi}{4} \right) (6.1)^2 = 18.4 \, \text{m}^2
\]

b) Maximum feed effective area: Average forward gain, \( g = 11.5 \, \text{db} \), 14.1; \( A_{e2} = \frac{g \lambda^2}{4\pi} = 0.008 \, \text{m}^2 \) for \( \lambda = 0.086 \, \text{m} \), that is, at 3.5 GHz.

c) The geometric mean effective area is \( A_e = (A_{e1} A_{e2})^{1/2} = 0.34 \, \text{m}^2 \)

d) Sensitivity for CassA, the brightest compact radio source: We will observe this first at 3.5 GHz. Expect about 620 Jy for the present flux at 3.5 GHz. For one polarization, the maximum \( T_{\text{ant}} = S(A_e k) = (620 \times 10^{-26})(0.34/1.4 \times 10^{23}) = 0.15 \text{K} \). For the 100 MHz bandwidth of the correlator, an integration time of 1 second, and a system temperature of 50 K, the output fluctuation will be:
\[
\Delta T = \frac{T_{\text{sys}}}{(BW \times \text{time})^{1/2}} = 0.005 \text{K}
\]

We get \( S/N = 30 \) in a one second measurement for the peak gain direction of the feed. If we use an integration time of 10 seconds for each pointing in the low gain directions, we will have \( S/N \sim 5 \) in those directions. In the forward directions, an integration time of 1-2 seconds should be adequate.

e) Sampling the beam pattern: With the feed moved to its most forward position, the pattern can be measured out to about 120° from the forward direction. Greater angles are shadowed by the edge of the primary reflector. From the theoretical pattern, 120° includes 95% of the feed energy. The beam pattern should be sampled at about 10° intervals for completeness. That will require about 300 pointings. At an average of 5 seconds per point, it will take one half hour. Allowing for slewing, it might take about three quarters of an hour. That should be manageable. The total feed gain is accurately known and averages 11.5 dB. To understand whether the unmeasured ~5% of the pattern in the back direction differs from the simulated pattern, we can compare the integral of the measured pattern out to ~120° with the measured total gain of the feed. Note that other feeds could also be mounted in place of the ATA feed and could therefore be measured.
IV. The effect of using a wideband signal to avoid some of the multipath problems.

The wideband signal correlates with itself strongly only over a limited delay. The relation is given by the Fourier transform.

\[
\int P(\nu) e^{2\pi i \nu \tau} d\nu = \int V(t) V(t+\tau) \, dt = c(\tau). \quad (2)
\]

\(P(\nu)\) is the power spectrum of the signal received at each antenna. The desired output signal is the correlation of the two signals, \(c(\tau)\), at zero relative delay(1). For a multipath signal, the relative delay is different from zero, and the extra path length reduces the correlation for the signal with the extra delay. Assume a low pass signal of upper band edge \(v_0\) and power level \(P_0\). The correlation function becomes

\[2\ P_0\ \{\sin 2 \pi v_0 \tau\}/2\pi\tau = c(\tau)\] \quad (3)

It is important to look at this familiar relation a little quantitatively, because when the feed is receiving from a low gain direction, an interfering signal may be being received from the direction of maximum feed gain. As noted above, the gain ratio may be as much as 26 dB (.05 in voltage). And \(c(\tau)\) does cut off sharply. Figure 2 shows the loss of correlation with extra path length \(d= c\tau\), where \(c\) is the speed of light, for extraneous reflected signals.
Figure 1. On the left is the image of the ATA antenna and feed. On the right is the result of removing the secondary mirror, the shroud, and the radome.

Figure 2. The correlation length $C(d)$ of the 100 MHz bandwidth signal as a function of extra distance $d$ in meters that a multipath signal would travel.