1hT memo 4

Drive equations for the Rapid Prototype Array antennas

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ABSTRACT

The 1hT Rapid Prototype Array antennas make use of an 'XY' mount which is unfamiliar to many astronomers. In this memo the drive equations are derived and investigated.

1. Introduction

The Rapid Prototype Array (RPA) antennas were manufactured by Orbitron (part SX-12), with an upgraded mount designed by Matt Fleming (and also manufactured by Orbitron). The basic mount geometry in each axis is of a triangle with one side fixed to the earth, one fixed to the antenna, and one of variable length (adjusted by an actuator). Two such arrangements are mounted one above the other to provide two-dimensional sky coverage. If the mount is positioned so that the lower actuator moves in a north-south plane, it can be thought of as a pseudo-equatorial mount: the lower actuator sets 'pseudo declination', while the upper actuator determines 'pseudo hour angle', providing motion in the sky at right angles to the meridan at the declination set by the lower actuator.[1]

2. Drive equations

Consider a triangle in a vertical plane with sides a, b and c (figure 1): a is the length of the side fixed to the earth, b is the length of the side fixed to the telescope (which points in the same plane as the triangle), c is the length of the variable (actuator) side, C is size of the angle opposite side c when the telescope is pointed at the zenith. The zenith angle due to this axis is α.

Then, from the cosine formula:

$$c^2 = a^2 + b^2 - 2ab\cos(\alpha + C)$$

[1] In a true equatorial mount the right ascension drive provides motion in the circle at constant declination, rather than as here, where the declination is constant only near the meridian.

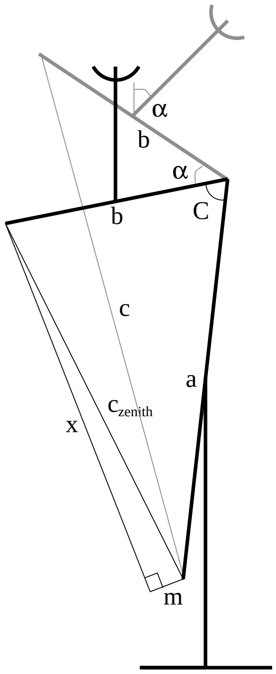


Fig. 1.— Functional diagram of a mount

But c is the hypotenuse of a right-angled triangle, where the actual actuator is another side (call the third, fixed, side m). So the actuator length, x, is given by

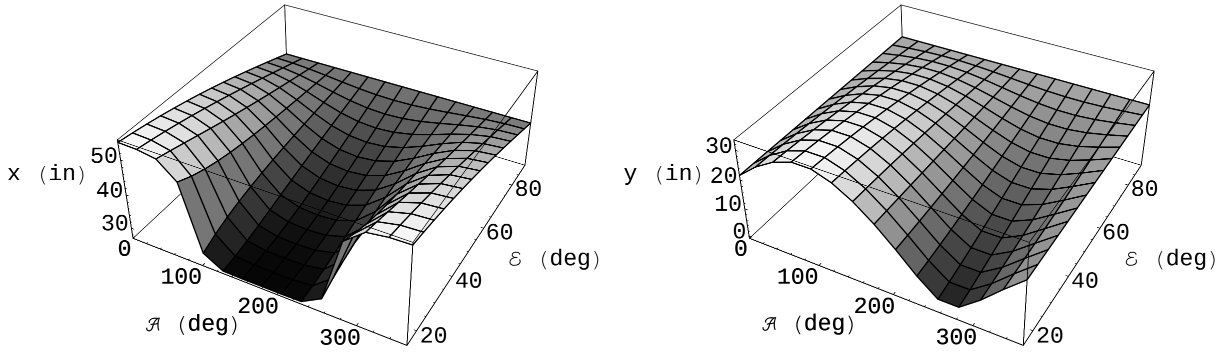
$$x^2 = c^2 - m^2.$$

Using β and y for the upper axis, and adopting subscripts to differentiate between the two axes, we have:

$$x = (a_x^2 + b_x^2 - 2a_x b_x \cos(\alpha + C_x) - m_x^2)^{\frac{1}{2}} \quad (1)$$
$$y = (a_y^2 + b_y^2 - 2a_y b_y \cos(\beta + C_y) - m_y^2)^{\frac{1}{2}} \quad (2)$$

From the spherical cosine and four-parts formulae we can relate α and β to the azimuth, $\mathcal{A}$, and zenith angle, $\mathcal{Z}$:

$$\begin{aligned} \sin\beta &= \sin\mathcal{A}\sin\mathcal{Z} \\ \tan\alpha &= \cos\mathcal{A}\tan\mathcal{Z} \\ \cot\mathcal{A} &= \sin\alpha\cot\beta \\ \cos\mathcal{Z} &= \cos\beta\cos\alpha. \end{aligned}$$



(a) X actuator

(b) Y actuator

Fig. 2.— Actuator lengths (inches)

Expressing the unknowns in terms of the desired azimuth and elevation, $\mathcal{E} = 90^\circ - \mathcal{Z}$, we obtain:

$$\beta = \sin^{-1}(\sin \mathcal{A} \cos \mathcal{E}) \tag{3}$$

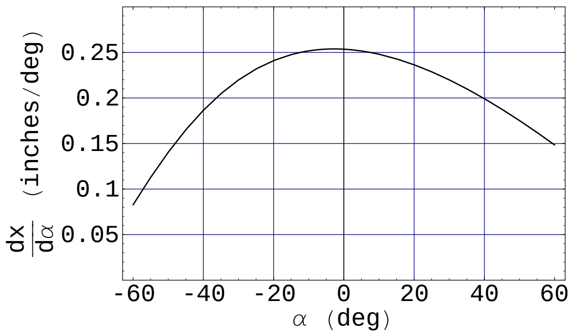
$$\alpha = \tan^{-1}(\cos \mathcal{A} / \tan \mathcal{E}). \tag{4}$$

3. Performance

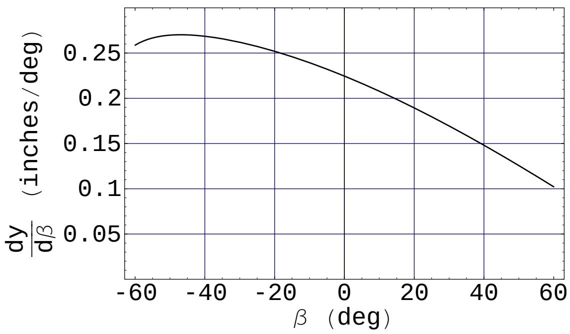
Figure 2 shows the actuator lengths required (from equations 1 and 2, and using the dimensions in table 3). Of more interest are the derivatives of the drive equations. These provide the ability to study the drive velocities required for tracking high-velocity objects and the encoder resolutions required to meet the pointing specification. In figure 3, plots are shown of the derivatives

a_x	42.190	in.
b_x	14.5	in.
C_x	1.26638	rad.
m_x	3.0	in.
a_y	19.017	in.
b_y	14.9	in.
C_y	1.38104	rad.
m_y	3.0	in.

Table 1: Dimensions of the RPA mount



(a) X actuator

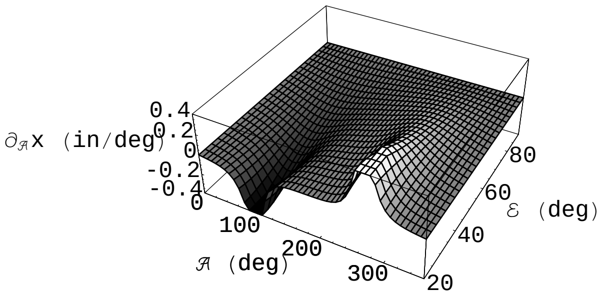


(b) Y actuator

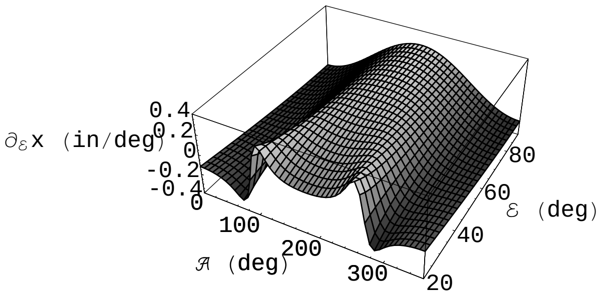
Fig. 3.— Actuator drive speeds for motion in the X and Y directions

of α and β (the sky angles due to each actuator) with respect to their corresponding actuator lengths. Note especially that the geometry of the mount makes tracking faster at lower elevations (for $|\alpha| \gg 0, \beta \gg 0$), while it also requires a higher encoder resolution for a given angular pointing specification.

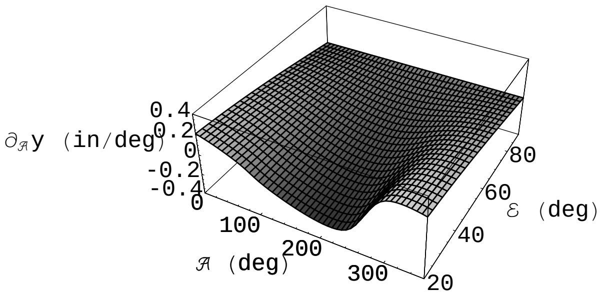
In Figure 4, the partial derivatives with respect to azimuth and elevation show us the required actuator velocities for driving in elevation and azimuth. Note the singularities at low elevation for $\mathcal{A} = 90, 270°$. Tracking sources will be more diffucult in these regions of the sky.



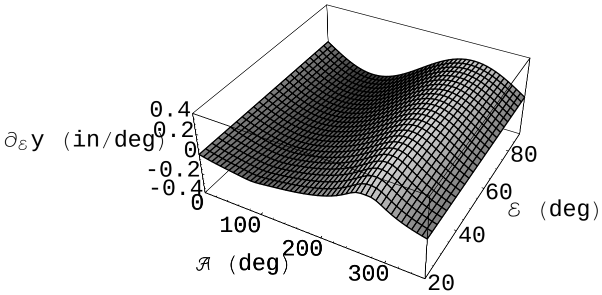
(a) X actuator (azimuth change)



(b) X actuator (elevation change)



(c) Y actuator (azimuth change)



(d) Y actuator (elevation change)

Fig. 4.— Actuator drive speeds for motion in the $\mathcal{A}$ and $\mathcal{E}$ directions (inches/degree)