ATA Memo No. 27

Notes on Delay and Phase Tracking for the ATA

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2001 May 09

Introduction

In an array telescope, whether used in beam-forming or in cross-correlation mode, it is necessary to equalize the signal delays among the various antennas from the direction of interest on the sky to the detector. Since the direction of interest is usually time-varying (if only due to earth rotation), the delay compensation must "track" that motion. Objects of interest may be extended, so that a range of directions is involved, in which case a "tracking direction" near the center of the range is defined. Or it may be desired to track several separate directions simultaneously; in this case, each must have its own signal processing channel with separate delay tracking.

Only the delay differences among antennas are important, but it is convenient to implement delay tracking separately for each antenna, with the required delay calculated relative to the signal arrival time at a reference location near the center of the array, where there may or may not be an antenna.

If the delay tracking were done in the RF part of the signal path, and if it were done perfectly, then no additional processing would be needed before detection. However, the delay tracking is actually done at baseband, and thus it affects not only the signal but also all previously-applied local oscillators; the result is a phase error proportional to the delay and to the net LO frequency, and this must be separately removed. In addition, the delay tracking is not perfectly accurate. It may have coarse resolution, where the error at any time is known; over a limited but substantial bandwidth, such small delay errors are similar to phase errors and can be compensated by small phase adjustments. Finally, it is difficult to ensure that the LOs are all absolutely in phase among the antennas, resulting in (mostly stable) phase differences among the baseband signals at the back ends. For interferometry, this can be corrected post-detection, but for beam forming it must be corrected in real time. All of these effects can be handled by making a time-variable phase adjustment at a single point in the signal path. This is called "phase tracking." The most rapidly varying component is due to the LO and is usually known as "fringe rotation." In many telescopes phase tracking is implemented in a local oscillator, but in the ATA it is desired to keep all LO circuitry simple, so phase tracking will be implemented as part of the digital signal processing.

In this memo, I give the numerical parameters for the delay and phase tracking required by the ATA. I do not attempt specify here any of the implementation methods or architecture.

Common Parameters

Maximum baseline (array size)	L	0.7 km initially 3 km ultimately
Maximum bandwidth of one signal channel	B	100 MHz initially 1 GHz ultimately
Frequency range of net LO		0.5 to 11.2 GHz
Sample quantization (word size)	w	8 bits

The "ultimate" values are arguable, and are given only to provide an upper limit on what might be required. The initial array's channel bandwidth of 100 MHz is not final, and may be somewhat more or less. The initial array's maximum baseline is based on ATA Memo 21.

Summary Of Results

	Initially	*Ultimately*	
Delay range (each baseband channel)	4.67	20.01	μsec
	7.5	320	kbits
Max delay rate, sidereal sources	8.49e-11	7.28e-9	
Max delay rate, design value (350 km LEO)	2.57e-8	1.10e-7	
Bandwidth for $< 2\%$ loss with no delay tracking	53.5	12.5	kHz
Min delay step interval, sidereal rate	11.8	0.28	sec
Min delay step interval, design value	39	0.91	msec
Coarse delay steps across antenna beamwidth	1.1	44.6	
Max phase rate, sidereal sources	0.951	4.075	Hz
Max phase rate, design value	287	1233	Hz

The "design values" are for a satellite in 350 km high circular orbit. A "coarse delay step" is one sample with dual quadrature Nyquist sampling.

Range Of Delay And Rate Of Change Of Delay

The maximum distance from an antenna to the central reference point is $L/2$, so the variable portion of the delay ranges from $-L/2c$ to $+L/2c$. Since negative delays cannot be realized, each antenna must support a variable delay of 0 to L/c. Additional (fixed) delay is needed to account for cable length differences among the antennas. In principle, only the outermost antennas need all of the variable delay and only the nearest to the central building need all of the fixed delay, but an implementation that is identical among the antennas is far more convenient, requiring each to handle the worst case. (It turns out that most of the delay can be implemented using inexpensive RAM, so the cost is determined mostly by the control and rate of change, rather than by the total delay.) We thus design for a delay range of 0 to $2L/c$.

The total data rate per channel is $2wB$, assuming Nyquist rate sampling (either 2 simultaneous samples are taken in phase quadrature at rate B, or one at rate $2B$). Thus the total storage required for the maximum delay is $4wBL/c$, which is 7.5 kbits for the initial array and 320 kbits ultimately.

The delay changes most rapidly as the tracking direction crosses the plane perpendicular to the line from the antenna to the reference position. For sidereal sources and a horizontal array, this occurs at meridian crossing and is maximum for antennas furthest east and west of the reference. The maximum rate is then $\dot{\tau} = \omega D/c$, where ω is the angular rate of motion of the tracking direction and D is the distance to the reference position. With $\omega = \omega_E$ for earth rotation (sidereal motion) and $D = L/2$, this gives 8.49e-11 (85 psec/sec) for the initial array and 7.28e-9 ultimately. But whereas we wish to track non-sidereal objects, including low-earth-orbit (LEO) satellites, angular rates many times this are possible. A maximum rate must be selected for the design. As a preliminary specification, consider a satellite in a 350 km high circular orbit *; in an overhead pass, its angular rate is .022 rad/sec, giving a worst-case delay rate of 2.57e-8 (initially, 1.10e-7 ultimately).

Any delay tracking implementation will have finite resolution $\delta\tau$, so the maximum rate is one resolution element in an interval $\delta\tau/\dot{\tau}$. We can assume $\delta\tau > 1/(10B)$, since finer resolution is not of significant benefit. This gives a minimum delay-step interval of $c/(10B\omega D)$, which is 11.8 (0.28) sec for sidereal objects and 39 (0.91) msec for 350 km LEOs, where the values corresopond to the initial (ultimate) array.

* 350 km is the lowest altitude of the International Space Station. Very few active satellites are lower, but one of them is the Shuttle, which is often as low as 200 km. Therefore, the tracking of 350 km LEOs is suggested as a requirement for the ATA, with 200 km as a design goal.

Effects Of Delay Error

Small errors in the delay (the order of a few reciprocal bandwidths or less) result in a loss of effective gain and thus of sensitivity. This is true for both interferometric and beam forming back ends. Whereas the delay is time varying with the tracking direction, the error tends to vary uniformly between a minimum and maximum centered around zero, $-\Delta\tau_{max}$ to $+\Delta\tau_{max}$. With a large array, the instantaneous errors among all the antennas tend to be uniformly distributed over the same range. A statistical treatment is therefore reasonable; the actual gain will be very close to its expected value.

Most back ends will analyze the channel into many narrow sub-bands, so it is worthwhile to calculate the gain as a function of frequency. The delay error is then equivalent to a linear phase error with frequency. We assume that the phase tracking has been separately adjusted so that, at the center of the channel's bandwidth, the net phase error is zero for every antenna; then there is no loss of gain at the center frequency. The phase error for one antenna is then $2\pi(f-f_0)\Delta\tau$ at frequency f and instantaneous delay error $\Delta\tau$, where f_0 is the center frequency. When the signals from all antennas are summed to form a beam, the result is

$$g(f) = \sum_{n=1}^{N} e^{j2\pi(f-f_0)\Delta\tau_n} = N\langle e^{j2\pi(f-f_0)\Delta\tau}\rangle = N\,\mathrm{sinc}[2(f-f_0)\Delta\tau_{\max}] \tag{1}$$

where the delay errors have been taken to be uniformly distributed among the antennas. This result is plotted in Fig. 1 for two cases: $\Delta\tau_{\max} = 1/2B$ and $1/4B$. Each corresponds to a delay resolution of one sample, the first for dual quadrature ("complex") Nyquist sampling and the second for single ("real") Nyquist sampling. This is rather coarse, resulting in significant gain loss at the band edges. Note that the value plotted is voltage gain; it should be squared to get power gain, which is then proportional to the detection SNR.

Fig. 1 also shows that if the bandwidth is sufficiently small, then large delay errors can be tolerated. If $\Delta\tau_{max} < 1/8B$, for example, the loss is less than 5% at the band edges and less than 1% for detection of the whole bandwidth. In the extreme case of no delay tracking at all, the delay error can be the propagation time across the whole array, $\Delta\tau_{\max} = L/c$. This produces negligible loss provided that $B < c/8L = 53.5$ kHz (12.5 kHz ultimately).

Even if the delay tracking is perfect, directions away from the tracking direction will have a delay error. It may be of interest to observe several directions within the antenna beam by using separate processing channels but a common delay tracker. To determine the feasibility of this, consider that for the furthest out antennas at the highest frequency, the antenna beam width corresponds to a delay range of about 1.1 samples (initial array, 700 m and 100 MHz; 44 samples ultimately).

Phase Tracking

As explained in the Introduction, there are several components to phase tracking. In general the hardware must synthesize, for each channel of each antenna, a phase function $\phi(t)$ that gives the total phase shift required at time t. Since the signal is discrete time, we require a new value at most every sample time, $t = t_i$. Let

$$\phi(t_i) = 2\pi f_L \mathbf{D}\cdot\hat{\mathbf{s}}(t_i)/c - 2\pi f_I \tau(t_i) + \phi_{\mathrm{cal}} + \phi_{\mathrm{null}}(t_i) \tag{2}$$

where f_L is the net LO frequency; $\mathbf{D}$ is the vector from the antenna to the reference location; $\hat{\mathbf{s}}$ is a unit vector in the tracking direction; f_I is the center frequency of the channel at the delay line (IF or baseband center); $\tau(t_i)$ is the actual delay currently applied by the delay tracker; ϕ_{cal} is the channel's absolute phase offset, normally due to the LOs, determined by calibration; and ϕ_{null} is an auxillary adjustment, for example to create a null in the array pattern in the direction of an interfering signal. The first term compensates for the lack of delay in the LO, and is simply the

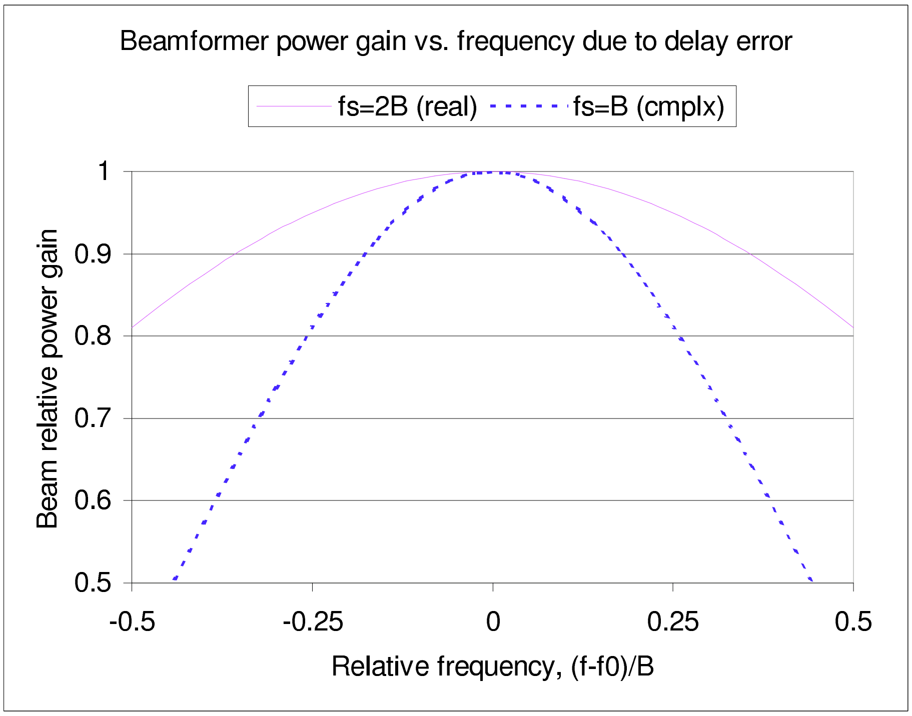

Figure 1 Power gain at beam center vs. frequency for coarse delay resolution. Delay errors at various antennas are assumed uniformly distributed.

geometrical signal delay times the LO frequency; the second is the phase shift caused by the delay tracker at mid-band, ensuring that the phase is correct at the band center in spite of any delay tracking error. The LO or geometrical term depends on the motion of the tracking position. For sidereal sources, it changes most rapidly on the meridian, for the most distant antennas, and at the highest observing frequency. This works out to be 0.951 Hz at 350 m distance and 4.075 Hz at 1500 m. However, LEO satellites move about 300 times faster, leading to phase rates exceeding 1 kHz.

The delay tracker correction term is constant while the delay remains constant. As calculated earlier, the delay tracking is typically constant for several seconds before stepping by one resolution unit, even if the resolution is fairly fine $(1/10B)$. For coarse resolution and/or for the inner antennas, the delay is sometimes constant for several minutes. Whenever a step change in delay occurs, a compensating step change in phase is needed here.

The calibration phase ϕ_{cal} corrects for the fact that the absolute phases of the LOs are not all equal; this correction is necessary to allow real-time beamforming. It must be stable by design, since it cannot be re-determined very often. It will change when the LOs are tuned to a new frequency or when some components are repaired or replaced, and it may drift slowly with environmental factors like temperature.

The auxiliary phase ϕ_{null} could be very rapidly changing, especially if it is intended to create one or more nulls in the array beam. If the nulling locations are rapidly moving, $\phi_{\mathrm{null}}(t)$ could be very complicated. We can think of the first term in (2) as allowing the array to follow the desired tracking position, while this term simultaneously tracks one or more other positions.