An Offset Gregorian Optical System for the Allen Telescope Array

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Abstract

This is a description of an offset Gregorian mirror system with a log-periodic feed as a candidate antenna for the ATA. The optical arrangement is designed to complement the wide band feed, providing the highest possible aperture efficiency over the 500 MHz - 11.5 GHz band. The challenging features of the feed are that its phase center moves with operating wavelength and that it has large spillover. The motion of the phase center requires a fairly large depth of focus, hence a large focal ratio. The other challenge for the reflector system is the need to operate at long wavelengths. The offset Gregorian is well suited for these challenges. The effects of defocus loss, spillover, illumination efficiency, edge diffraction, gain loss due to surface roughness, and shielding against ground spillover are included in the discussion. An example of an overall feed and mirror solution is discussed in detail.

1 Introduction

The plan for the Allen Telescope Array is for its collecting area of 10^4 m^2 to be the result of a large number of small antennas with their outputs combined in an array. Economic considerations argue that the optimum size of each primary reflector should be of the order of 5m, and the present study is based on a reflector of that size. A second constraint is that an inexpensive MMIC input amplifier with a low noise temperature can be constructed which has an instantaneous bandwidth covering 500 MHz to 11 GHz(Weinreb, private communication). The most economic and flexible arrangement is to develop an associated antenna with feed that operates over that same instantaneous band. That large bandwidth is no problem for the reflector part of the antenna, but it requires use of some member of the class of frequency independent antennas for the feed. After some experimentation (largely by computer simulation), a high gain ($\sim$ 11.5db) filled zig-zag has emerged as a suitable candidate. The specified goal for the optical system is for aperture efficiency of 0.5 or better over 1-10 GHz and for operation with only slight degradation in efficiency over 500 MHz to 11.5 GHz.

The three main problem areas for this study are (1) that the feed focus moves with wavelength, (2) that the feed spillover onto the ground is a potential source of high background noise, and (3) that the operation to wavelengths as long as 60cm is degraded by edge diffraction effects for such a small reflector. Consideration of these problems leads to the suggestion of a particular choice of antenna system which minimizes them but may be a little awkward to construct. This system is an offset Gregorian.

2 Properties of the Feed

To achieve the large bandwidth of 500 MHz - 11 GHz, which the PHEMT amplifier enables, a feed which is in the class of frequency independent antennas has been chosen. The choice, discussed separately, is a zig-zag log-periodic antenna. It's expansion angles in both angular coordinates are 20^o, its log period is .975, and it is a filled structure for minimum losses. Its properties have been studied with an electromagnetic simulation computer package, and first order tests on a constructed prototype verify its computed characteristics. The following summary properties are based on the simulations.

- The E- and H-plane patterns are nearly the same in the forward lobe, and the magnitude of the approximately symmetric electric field to the first minimum at about 66^o is given by

  $$E = 1.000 - .000754\theta^2 + 2.28 \times 10^{-7}\theta^4 - 2.42 \times 10^{-11}\theta^6 \quad (1)$$

  θ is the polar angle in degrees. This pattern is essentially independent of frequency over the operating band.

- The feed gain is 11.5 db and the ohmic losses are in the range $2-4\%$.
- Both linear polarizations are received. This is possible because the two expansion angles are the same 20^o. The two antennas for the two polarizations are on adjacent faces of a pyramid.
- The input impedances of the two polarizations are about 220 ohms and are balanced.
- The center of phase of the emission in each polarization is 1.4 wavelengths from the origin of the antenna figure. The input terminals are at 1.7 cm from the origin. The shift of the phase center with wavelength is a negative feature of the antenna for its use as a feed for a reflector system. A relatively long focal ratio with large depth of focus is necessary for this feed for operation over a large range of frequencies.
- Unlike horn feeds, the log-periodic feed tends to have large side-lobes and back lobes. Special precautions are required to make certain that these lobes are not terminated in the ground, which would raise the noise temperature of the system.

3 General Reflector Considerations

The simplest reflector arrangement would be a prime focus system, either symmetric or offset. The moving feed phase center with wavelength requires a large focal ratio for adequate depth of focus for the large wavelength range. Coupling this situation with the large side and back lobes of the log-periodic feed results in considerable spillover to the ground. A screen at the edge of the reflector would need more collecting area than that of the reflector to lower the spillover adequately.

A symmetric Cassagrain arrangement puts much of the spillover onto the sky, at least at high elevations, and would be an important improvement. The rule-of thumb that a mirror should be 4-5 wavelengths in diameter not to have excessive diffraction losses requires that the secondary mirror be at least 2.4m in diameter to operate to the longest wavelength of 60cm. A 2.4m secondary for a 5m primary represents a geometrical blockage of 23%. Since the blockage is in the center of the aperture, a primary diameter of about 6m would be necessary to recover the lost area. Shaping the mirrors so that the rays from the feed are all reflected past the secondary eliminates the signal blockage but not the loss in area. Also, the hole in the central illumination results in high sidelobes in the main pattern. Furthermore, the geometrical shaping may not work well at the low frequencies.

A further step is to make the secondary offset. While this helps with primary dish size and the main beam sidelobes, the back radiation of the feed is no longer caught by the primary and sent onto the cold sky. The feed must be shielded to avoid the spillover onto the ground. Shielding the feed rather than the primary reflector results in a smaller shield.

An offset Gregorian rather than an offset Cassagrain appears to be a better choice, because it suggests a simple way to add the shield for the feed. Continuing the secondary ellipsoid around the back of the feed will cause all the spillover rays to go onto the sky for elevation angles of 0 to 90^o if the Gregorian mirror is placed below the primary. Figure 1 shows a view of this optical arrangement. The top of the shield may be covered with a transparent plastic plate. This, along with the shield, will provide a total environmental cover for the feed. Based on these advantages, this system is the choice for the present study, but mechanical mounting considerations may yet require one of the simpler systems above as a compromise.

4 Gain Loss Due to Movement of the Feed phase Center,η_d

The phase center of the feed is normally located at the focus of the reflector to ensure a uniform phase across the reflector aperture. For a stationary log-periodic feed, the phase center position is different at different wavelengths, and the system can be in focus at only one wavelength.

For the 20^o Zig-Zag that we have studied in detail, the phase center of the radiation pattern is located 1.4λ from the convergence point of the log-periodic structure. A lower gain feed will have a less displaced phase center but lower gain and a fatter pattern. A lower focal ratio may be used for a given bandwidth, but that effect will be canceled by the fatter pattern of the lower gain antenna. A preliminary study of this trade-off, based on historical log-periodic antenna studies, suggests that the overall spillover is less for the highest gain feed. A more accurate study of this trade-off based on simulations is planned to find if there is an optimum feed gain with respect to this effect.

The effect of an axial focus error is a quadratic phase error across the antenna aperture. This error distribution can be converted to an effective RMS phase error and the corresponding gain loss can be calculated. Figure 2 shows the result of such a calculation. The quadratic dependence on focus is evident as is the dependence on focal ratio F =f/D. There is also a weak dependence on the edge illumination. In practice, the system is focused at one wavelength near the middle of the band, and F is chosen so that the gain loss is acceptable at the ends of the band.

5 Illumination Efficiency, η_{Ill}

One of the factors that affects the net aperture efficiency is the illumination efficiency. This is mostly just a function of the level of the illumination at the aperture edge. It is the following familiar integral.

$$\eta_{Ill} = \frac{2\pi |\int_0^R E(r) r dr|^2}{\pi R^2 \int_0^R |E(r)|^2 r dr} \tag{2}$$

r is the radial variable in the aperture, and E(r) is the electric field pattern, eq(1), transformed to the aperture coordinate, r. R is the radius of the aperture. The integral has been carried out for a range of fractional edge electric fields, E_0, and uses the log-periodic feed field distribution of equation (1). The result is

$$\eta_{Ill} = 0.3656 + 3.7324E_0 - 8.8208E_0^2 + 7.5836E_o^3 \tag{3}$$

This is a generally useful result, since main beam pattern shapes are mostly the same. η_{Ill} is also calculated as a function of the focal ratio, F, which is a result which is particular to the present feed.

$$\eta_{Ill} = -0.5425 + 3.8971F - 3.3967F^2 + 1.00233F^3 \tag{4}$$

Depending on what value of F is chosen, the illumination efficiency can be calculated from equation 4.

The following table shows a few values for the above equations.

F_{eff}	E_o	η_{Ill}
0.5	0.114	.683
0.6	0.139	.789
0.7	0.254	.866
0.8	0.343	.914

6 Spillover efficiency, η_{sp}

Whatever the choice of the angle, θ_e, which corresponds to the edge of the reflector in the polar coordinates of the feed, only a fraction η_{sp} of the power radiated by the feed will be collected by the reflector. The rest, side and back lobes, will be lost. Corresponding to that angle there is a focal ratio, F, given by

$$F = 1/4\sqrt{\frac{1 + cos\theta_e}{1 - cos\theta_e}} \tag{5}$$

The expected focal ratio will be in the range .60 to .80 for the best compromise between the spillover and the loss due to the out of focus condition. Hence, it is useful to have a simple formula that gives the spillover efficiency as a function of F for the simulated pattern over the above range. The result is

$$\eta_{sp} = .8323 + .1130F - 0.3537F^2 \tag{6}$$

which is computed for the range $.59 \leq F \leq .87$.

7 Surface Roughness Effects

The effects of imperfections in the surface can be evaluated if the RMS error of the surface is known. For a surface RMS error of σ and a wavelength λ, the loss in gain is given by the famous Ruze factor

$$exp - [\frac{4\pi\sigma}{\lambda}]^2 \tag{7}$$

One of the antenna manufacturers claims to be making 5m class surfaces with an RMS error of .025 inches, or 0.64mm. For antennas in the 2m class, the claim is .008 inches or 0.2 mm. These errors only lower the gain at the upper end of the band by a few percent.

8 Design of the Dual Reflector Optical System

Detailed design of the dual reflector mirror system allows an optimum arrangement in which one secondary focus position is chosen for best gain and minimum cross polarization (Rusch et al, 1990). For this choice, there is an equivalent parabola, and a symmetrical primary feed pattern is transformed into a symmetrical final pattern through the optical system. The choice of parameters for this application is enormously assisted by a program with the name RASCAL developed by the Applied Electromagnetics Group at USC (Lee, Brown, and Prata, 1995). This program allows one to select primary and secondary mirror sizes and positions and then gives the optimum layout in a useful plot and table.

Figure 1 shows an off-set Gregorian optical system as developed by RASCAL that connects well to the log-periodic feed. The primary and secondary mirrors are shown as solid curves. The dotted line is an extension of the secondary ellipsoid which serves to keep all of the feed spillover radiation from reaching the ground, All rays from the feed which are reflected from the ellipsoid pass through the secondary focus. After that, they either move toward the sky directly or are reflected in that direction by the primary mirror. Evidently, rays which leave the focus of the feed and do not intersect the extended secondary also pass on toward the sky. Figure 1 corresponds to the telescope pointing toward zero elevation. For all elevations between 0 and 90^o, the scattered radiation is also directed toward the sky. This is an important feature, since about 25% of the feed radiation misses the Gregorian secondary. There should be no ground contribution to the system temperature from geometrically scattered radiation. Whereas the surface accuracies of the primary and secondary reflectors must correspond to RMS errors of the order of 0.5mm, the extension of the ellipsoidal secondary can be a surface of much poorer accuracy.

9 Edge Diffraction Effects

The sharp edge cutoff in the aperture illumination produces diffraction which has three principal effects. (1) The main beam pattern has sidelobes, which are familiar features of long wavelength telescopes. (2) Especially when the apertures are only a few wavelengths in diameter, the edge diffraction decreases the effective area of the aperture. (3) This same diffraction loss produces scattering to the ground which, in turn, increases the system temperature. The latter two effects are important in the present design, because the goal is to operate at wavelengths as long as 60cm with a primary mirror that is only about

5m in diameter.

The effects of diffraction on the aperture efficiencies in Cassagrain antenna systems are discussed by Kildal (1983). He gives simple formulas for the efficiency corrections for these two-mirror systems. Because the effects are essentially edge effects, the formulas should apply equally to Gregorian systems. In the model discussed here, the offset Gregorian is optimized to produce a symmetric pattern, and we assume that the average edge diffraction around the beam will be the same as in Kildal's axially symmetric calculations. The only change that we make is to omit the correction term corresponding to the blockage of the primary by the secondary. The additional factor due the the edge diffraction is

$$\eta_i = [1 - C_d E_o \sqrt{(\lambda/d)}\sqrt{(1 - d/D)}]^2 \equiv [1 - l_d]^2 \tag{8}$$

d is the secondary diameter, and D is the primary diameter. E_o is the fractional edge electric field. η_i is the efficiency factor due to diffraction. l_d is defined by equation (8).

$$C_d = \frac{cos^2(\phi_r/2)}{\pi\sqrt{sin(\phi_v)}} C_b \tag{9}$$

ϕ_r is the half angle of the secondary, and ϕ_v is the half angle of the primary as viewed from its focus. C_b is a number which depends on the edge illumination of the mirrors. It is given in Kildal's Figure 4 and is about 1.5 for an edge illumination of -10 db and a little larger for lower edge illumination. For given values of d and D the wavelength dependence of the correction is $\sim \sqrt{\lambda}$, which is gradual.

Whatever the loss in aperture efficiency, the diffracted radiation tends to go in all directions. That is because the diffraction is essentially an edge effect and each piece of the edge scatters the radiation as if it were part of an infinite knife edge (cf Morse and Feshbach, 1953, vol II, p 1385). Thus, we assume that half of the power, l_d, goes toward the ground regardless of the antenna orientation. This ground coupling will add to the system temperature.

10 A straw man Design for a 5m Offset Gregorian

As a conclusion, here is a summary straw man design of an offset Gregorian which includes all of the effects listed above. Figure 1 is an approximate representation of the design. The operating frequency range is 500 MHz to 11.25 GHz. The primary mirror is 5m in diameter; the secondary is 2.4m. The system is in focus at 6.25 GHz, the focal ratio at that frequency is 0.65, and the edge illumination is -13.5 db ($E_o = .211$) at that frequency. Because the phase center of the feed is 1.4λ, the system goes out of focus at other frequencies, and the edge illumination also varies. It increases at lower frequencies where there is an improvement in illumination efficiency but greater diffraction losses and greater spillover. The ellipsoidal secondary wraps around the feed, so that about 2/3 of its diffraction scatter to the ground is eliminated. The ground pickup from the edge diffraction is calculated from 2/3 l_d. ($1/2 + 1/3 \times 1/2 = 2/3$.) There is additional spillover at the low frequencies due to the feed being out of focus there. This spillover goes past the primary after reflection of the feed signal from the secondary in the out-of-focus setting at low frequencies. This effect produces higher system temperatures and lower gain at the lowest frequencies. The Ruze surface scattering assumes that the surface RMS is .64mm for the primary and negligible for the secondary.

The following tables give the contributions from the various effects discussed above for a selection of frequencies over the range 500 MHz to 11.25 GHz. η_T is the overall aperture efficiency, and the last column gives the amount of radiation that is scattered to the ground. The first table gives the result for a fixed focus as appropriate for 6.25 GHz. In this case $\eta \geq 0.5$ for $1 \leq \nu \leq 10 GHz$. The second table gives the result in the case that the feed be at the correct focal position at each frequency.

11 Fixed Focus Offset Gregorian

ν	η_{def}	η_i	η_{sp}	η_{ILL}	Ruze	η_T	Gnd Spill
11.25	0.85	.986	.820	.832	.915	.52	1.6%
10.0	0.91	.985	.820	.832	.932	.57	1.6%
8.75	0.985	.984	.820	.832	.947	.62	1.7%
7.50	0.99	.982	.817	.832	.961	.64	1.7%
6.25	1.00	.981	.817	.832	.973	.65	1.7%
5.00	.99	.978	.815	.838	.983	.65	1.7%
3.75	.985	.974	.812	.842	.990	.65	1.9%
2.50	.91	.953	.806	.851	.996	.59	2.5%
1.25	.89	.945	.791	.863	1.00	.57	3.8%
1.0	.84	.942	.758	.867	1.00	.52	5.2%
.75	.82	.922	.724	.893	1.00	.49	7.3%
.50	.805	.888	.650	.914	1.00	.43	9.5%

12 In Focus Offset Gregorian

ν	η_{def}	η_i	η_{sp}	η_{ILL}	Ruze	η_T	Gnd Spill
11.25	1.00	.986	.817	.832	.915	.61	1.4%
10.0	1.00	.995	.817	.832	.932	.62	1.5%
8.75	1.00	.984	.817	.832	.947	.63	1.5%
7.50	1.00	.982	.817	.832	.961	.64	1.5%
6.25	1.00	.981	.817	.832	.973	.65	1.6%
5.00	1.00	.978	.817	.832	.983	.65	1.7%
3.75	1.00	.975	.817	.832	.990	.65	1.8%
2.50	1.00	.969	.817	.832	.996	.65	1.9%
1.25	1.00	.957	.817	.832	1.00	.65	2.3%
1.0	1.00	.952	.817	.832	1.00	.65	2.5%
.75	1.00	.944	.817	.832	1.00	.64	2.7%
.50	1.00	.932	.817	.832	1.00	.63	3.1%

13 Bibliography

1. Lee, Y., Brown, K. W., Prata, A., 1995 USC copyright
2. Rusch, W., Prata, A., Rahmat-Samii, Y., and Shore, R. 1990, IEEE, Trans. AP, AP-38, No. 8.
3. Kildal, P-S., 1983, IEEE, Trans. AP, AP-31, No. 6

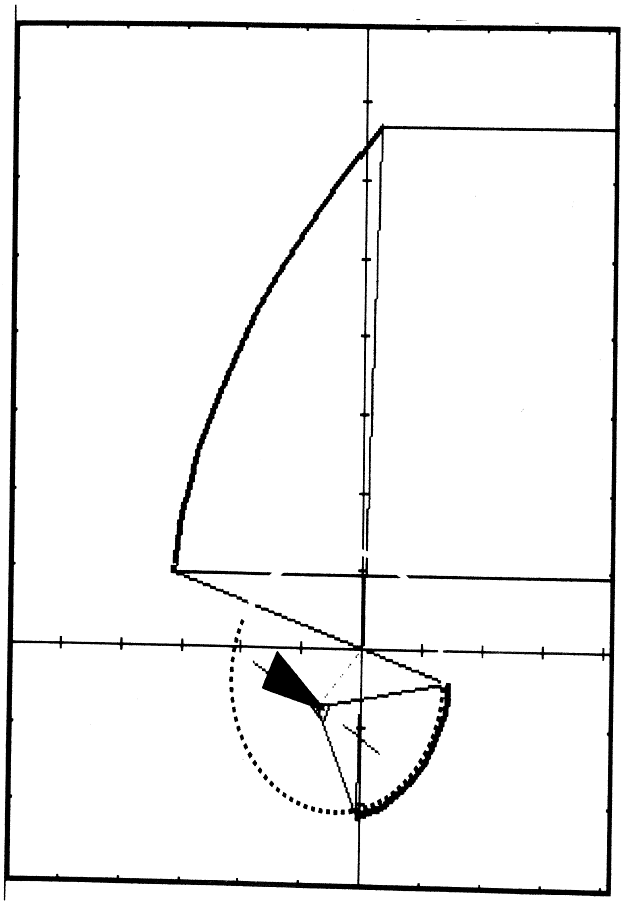

Figure 1: An offset Gregorian optical system viewed in cross section. The exit window is 5m in diameter; the diameter of the ellipsoidal secondary, the solid curve, is 2.4m. The log-periodic feed is the filled triangle. The dotted curve is an extension of the ellipsoidal feed to shield side and back lobes of the feed from the ground. The orientation of the system is toward an elevation angle of 0^o with the secondary below the primary.

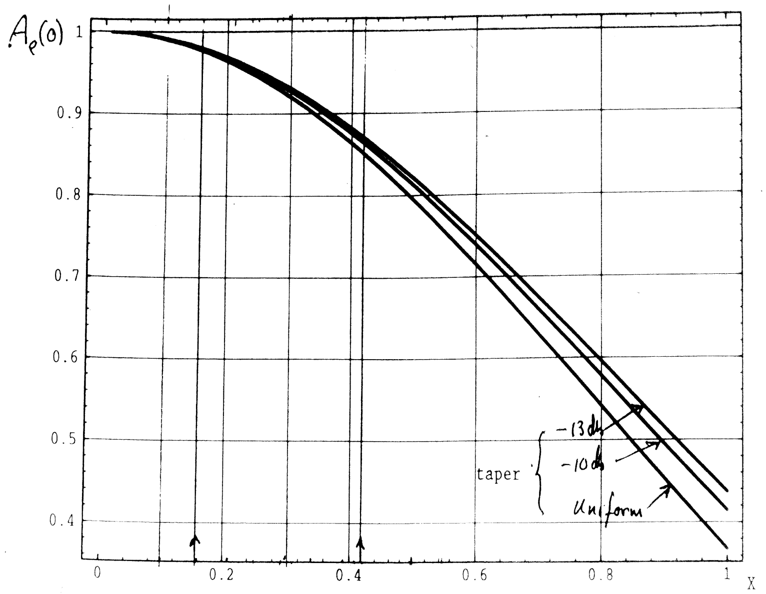

Figure 2: Gain loss due to axial focus error s, where $X = (17)(s/\lambda)(1 + 16(F/.41)^2)$. The three curves are for different edge tapers (illumination).