Beam Forming and RFI Elimination with the 1HT

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1 Forming the Beam

One goal is to add the outputs of all the antennas for some finite band to form a beam in some direction on the sky. This will be the case for SETI observations. The system downconverts this piece of the RF band of each antenna to baseband and then adds some delay. All these signals are then simply added. The output for the peak of the beam should be about N times the voltage that arrives at each antenna, where N is the number of antenna elements. The system forms a beam in a desired direction (possibly) fixed among the stars and removes the Doppler shifts and delay changes associated with the Earth's rotation. The control scheme works equally well for either beam formation or cross-corelation.

One antenna serves as the reference. The others are located at distances $\vec{S_i}$ with respect to the reference antenna. Consider the formation of a beam for a bandwidth ν_m. The band is adjacent to the frequency ν_0 and is converted to baseband by an oscillator at that frequency, that is, to $0 \leq \nu \leq \nu_m$. ν_0 may be a single converter frequency, or it may be the net conversion frequency for an up/down converter.

The voltage arriving at the reference antenna from the distant source is v(t).

$$v(t) = \int_0^\infty a(\nu) cos(2\pi\nu t + \phi(\nu))\, d\nu \tag{1}$$

Signals arriving at the other antennas are the same except for the time delays associated with their distance from the reference antenna. $a(\nu)$, the spectrum of v(t), is set to zero outside the band $\nu_0 \leq \nu \leq \nu_0 + \nu_m$ by a filter. We assume that the distant source is a point source, so that the signals arriving at all the antennas differ only by geometric delays. For antenna i,

$v_i(t) = v(t - \tau_{ig})$, where the geometric delay is given by the following.

$$\tau_{ig} = -\vec{S}_i \cdot \hat{n}(t)/c \tag{2}$$

$\hat{n(t)}$ is the instantaneous direction of arrival, constantly changing due to the Earth's rotation.

$$v_i(t) = \int_0^\infty a(\nu) cos(2\pi\nu(t - \tau_{ig}) + \phi(\nu))\, d\nu \tag{3}$$

The block diagram for the circuit is as follows.

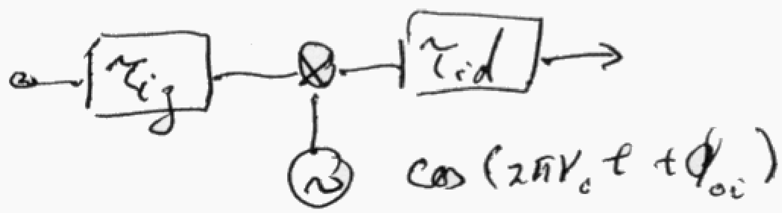

Figure 1

$\omega_0 = 2\pi\nu_0$ translates the band to baseband. ν_l is the low frequency variable, $\nu_l = \nu - \nu_0$.

$$\phi_{0i} = \int_0^t \omega_{0i}(x)\,dx + C_i + \psi_i(t) \tag{4}$$

C_i are the phase constants needed to zero all the instrumental phases for the required beam direction on the sky at t=0. They can be found by observing a bright point source calibrator in advance of other observations. ψ_i is a slow phase switching, e. g., a Walsh function, which may be needed to zero DC drifts. The phase (4) can be added in any mixer in the overall chain.

The mixer multiplies the two signals, which results in the difference between the two phases of the cosine functions. The total output phase is now

$$2\pi\nu(t - \tau_{ig}) - 2\pi\nu_0 t - \phi_i(t) = 2\pi\nu_l t - 2\pi\nu_0\tau_{ig} - 2\pi\nu_l\tau_{ig} - \phi_i(t) \tag{5}$$

To correct the phase for frequencies other than ν_0, those near ν_0 that end up in the band $0 \leq \nu_l \leq \nu_m$, we now introduce a delay τ_{id} in the video band. Then the total phase term is:

$$2\pi\nu_l(t - \tau_{id}) - 2\pi\nu_0\tau_{ig} - 2\pi\nu_l\tau_{ig} - \int_0^t \omega_{0i}(x)\,dx - C_i - \psi_i(t) \tag{6}$$

The different terms can be usefully grouped as follows.

$$= 2\pi\nu_l t - [2\pi\nu_l(\tau_{ig} + \tau_{id})] - [2\pi\nu_0\tau_{ig} + \int_0^t \omega_{0i}(x)\,dx + C_i] - \psi_i(t) \tag{7}$$

Let τ_m be the maximum delay of the delay line. Then set

$$\tau_{id} = \tau_m/2 + \vec{S_i} \cdot \hat{n}_0/c, \tag{8}$$

where $\hat{n_0}$ is the direction of the peak of the array beam. For the general direction $\hat{n}$,

$$\tau_{ig} + \tau_{id} = -\vec{S_i} \cdot \hat{n}/c + \tau_m/2 + \vec{S_i} \cdot \hat{n}_0 \tag{9}$$

This simplifies to the following.

$$\tau_{ig} + \tau_{id} = \tau_m/2 - \vec{S_i}/c \cdot (\hat{n} - \hat{n}_0) \tag{10}$$

The second term in the square brackets in (7) is thus $2\pi\nu_l\tau_m/2$, corresponding to a constant delay, when $\hat{n} = \hat{n_0}$.

Now zero the fringe rate with respect to the reference antenna by zeroing the third term in brackets in equation (7). This requires that

$$2\pi\nu_0\tau_{ig} + \int_0^t \omega_{0i}(x)\,dx + C_i = 0 \tag{11}$$

We satisfy this condition for the peak direction, $\hat{n}_0$, by taking the time derivative of equation (11).

$$2\pi\nu_0\frac{d}{dt}(\tau_{ig0}) + \omega_{0i}(t) = 0 \tag{12}$$

Thus the formula for the offset frequency is

$$\omega_{0i}(t) = 2\pi\nu_0\frac{d}{dt}[\vec{S}_i\cdot\hat{n}_0]/c \tag{13}$$

For other directions, $\hat{n}$, the third term in the brackets has some residuals.

$$-2\pi\nu_0[-\vec{S}_i\cdot\hat{n}]/c - \int_0^t [2\pi\nu_0\frac{d}{dt'}(\vec{S}_i\cdot\hat{n}_0/c)]\,dt' - C_i \tag{14}$$

which becomes

$$= 2\pi\nu_0/c\vec{S}_i\cdot(\hat{n}-\hat{n}_0) - C_i \tag{15}$$

Leaving out the terms C_i and $\psi_{0i}(t)$, we now find for the total phase

$$2\pi\nu_l t - 2\pi\nu_l\tau_m/2 + 2\pi\nu_l/c\vec{S}_i\cdot(\hat{n}-\hat{n}_0) + 2\pi\nu_0/c\vec{S}_i\cdot(\hat{n}-\hat{n}_0) \tag{16}$$

Combining terms, we have

$$2\pi\nu_l(t-\tau_m/2) + 2\pi\nu/c\vec{S}_i\cdot(\hat{n}-\hat{n}_0) \tag{17}$$

We now have the total expression for the output voltage of the ith antenna following the down conversion and the addition of delay at the baseband.

$$\int_0^\infty a(\nu)cos[2\pi\nu_l(t-\tau_m/2) + 2\pi\nu/c\vec{S}_i\cdot(\hat{n}-\hat{n}_0) + \phi(\nu) - C_i - \psi_{0i}(t)]\,d\nu \tag{18}$$

Now we add up all the voltages with the weights w_i, which are selected to produce the desired beam pattern. At this point, we can leave out the phase constants C_i, which are determined by calibration, and the phase switching terms ψ_i, which are probably not needed for the beam formation. The total is

$$O(t) = \sum_{i=1}^{N} w_i \int_0^\infty a(\nu) cos[2\pi\nu_l(t-\tau_m/2)+2\pi\nu/c\vec{S_i}\cdot(\hat{n}-\hat{n}_0)+\phi(\nu)]\, d\nu \quad (19)$$

At the peak of the beam, $\hat{n} = \hat{n}_0$, and

$$O_0(t) = \sum_{i=1}^{N} w_i \int_0^\infty a(\nu) cos[2\pi\nu_l(t-\tau_m/2)+\phi(\nu)] d\nu \quad (20)$$

The integral in (20)is just the original voltage arriving at the reference antenna, shifted to baseband and delayed by $\tau_m/2$, which we can call $\hat{v}(t-\tau_m/2)$. Since $\sum_{1=i}^{N} w_i \sim N$,

$$O_0(t) = N\hat{v}(t-\tau_m/2) \quad (21)$$

Thus, we get the expected result, that at the center of the beam we just get N times the voltage that arrives at one antenna. In directions other than the beam center, the frequency dependence of the beam shape distorts the spectrum of the received signal. The complex array pattern, $E(\nu,\theta,\phi)$, is

$$E(\nu,\theta,\phi) = \sum_{i=1}^{N} w_i e^{i2\pi\nu/c\vec{S_i}\cdot(\hat{n}-\hat{n}_0)} \quad (22)$$

where θ and ϕ are polar angles about the beam center $\hat{n}_0$. A little bit more algebra shows that in a general direction in the beam the output is

$$O(t) = \int_0^\infty a(\nu) E_r(\nu,\theta,\phi) cos[2\pi\nu_l(t-\tau_m/2)+\phi(\nu)] d\nu \quad (23)$$

$$- \int_0^\infty a(\nu) E_i(\nu,\theta,\phi) sin[2\pi\nu_l(t-\tau_m/2)+\phi(\nu)] d\nu \quad (24)$$

The subscripts r and i stand for the real and imaginary parts of E.

2 Summary of the Control

At each antenna, the signal is translated to baseband by an oscillator of net frequency ν_0.

- At some point in the chain, an offset frequency, ν_{0i} must be added.

$$\nu_{0i} = \nu_0 \frac{d}{dt}[\vec{S}_i \cdot \hat{n}_0] \tag{25}$$

- Delay must be added in the video band.

$$\tau_{id} = \tau_m/2 + \vec{S}_i \cdot \hat{n}_0/c \tag{26}$$

- Fixed phase offsets for all the antennas must be found by a calibration observation of a point source.

3 Elimination of RFI

Here is a possibility for elimination of RFI; there may be other schemes that are more effective or simpler. These ideas can be tested with the Rapid prototype Array.

3.1 A Cancelation Scheme

Possibly the worst interference sources will be the communication satellites. However, their positions should be acurately known as functions of time, which is a help. The idea here would be to form a second beam (simultaneous with the main observing beam) directed at the interfering satellite. It would use the control scheme discussed above and would operate on a set of baseband electronics which is the same as that of the main beam. For the beam directed at the satellite, the phase and delay settings are given by equations (25) and (26) respectively, but the vector $\hat{n}_0(\mathrm{t})$ is now $\hat{n}_{sat}(\mathrm{t})$, which corresponds to the orbit of the satellite. The array now has a wide-band beam directed at the satellite, in addition to the main observing beam. Figure 2 shows a block diagram of the sets of phase and delay networks for both the main beam and a number of satellite directed beams.

By itself, the array beam directed toward the satellite would produce a perfect measurement of the satellite's radiated voltage toward the array. On the other hand, the dishes are pointed toward the main beam direction, and the satellite measurement is made in the direction of sidelobes of the dishes. This will produce some dispersion in the satellite signal. Of course, this dish distortion of the satellite signal is the same that occurs in the reception of the satellite signal in the main beam. Let us label the satellite signal distorted only by the dish sidelobe pattern $\hat{v}_{sat}(t)$. It is what we obtain from the measurement with an array beam directed at the satellite. Its Fourier Transform is calculated to be $\hat{a}_{sat}(\nu)$. The satellite interference in the main beam can now be calculated from equation (23,24) with $a(\nu)$ set equal to $\hat{a}_{sat}(\nu)$. Note that the effect of the dish pattern cancels out. If we assume that the array pattern is exactly known, the voltage found by putting $\hat{a}_{sat}(\nu)$ in (23,24) should be simply subtracted from the main beam time signal to eliminate the interference. This final operation, the calculation of equation (23,24), will have some delay associated with it. It is the same sort of calculation that is done for the SETI spectrometer. This delay can be compensated by adding it to each of the main beam forming delay lines. Note that a number of such satellite measurements and corrections can be done at the same time for other satellites in the same band. GPS, for example, with a bandwidth of about 20MHz at 1.62 GHz, typically has about eight LEO satellites up at the same time.

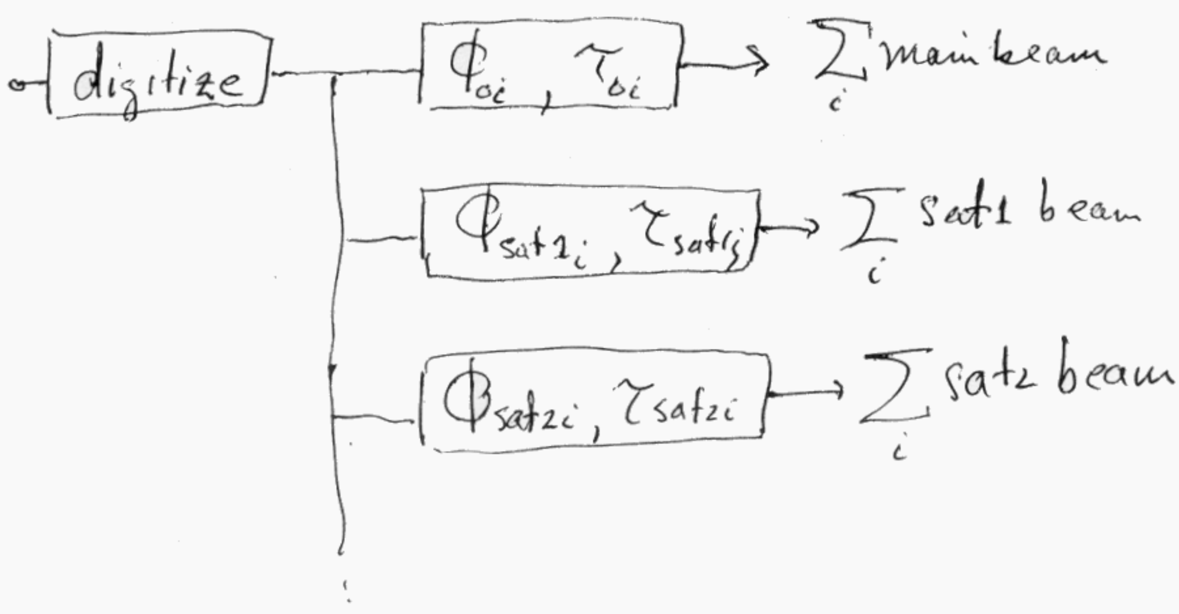

Figure 2