Forming a Broadband Null with the ATA Using the Tree Algorithm

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1 Introduction

In a previous memo (Dreher and Welch, 1999), we discussed the process of forming a beam with the ATA which could be pointed toward a position on the sky which was either fixed in direction or moving. That memo also showed how the array could be made blind to signals from the directions of satellites, so that observations could be made for SETI and other signals in the bands where the satellites broadcast. That scheme was based on the idea that, simultaneous with forming a beam in some desired direction, a second beam could be directed toward the satellite to measure its signal and subtract its interference in the main beam signal. It's a very simple idea which looks practical. An alternative way of dealing with the interference is to form a null in the direction of the satellite, so that no interfering signal is received at all and no subtraction need be made. The final effect is the same, but one approach may be more effective than the other. Earlier papers have described methods for introducing nulls (Rapaport, 1995), but they have all been restricted to very narrow band signals. This memo describes one scheme to introduce a broadband null in the array pattern.

2 The Broadband Null

This scheme follows the Tree method, which has been successfully applied to narrow band, closely spaced arrays. The basic idea is that the outputs of adjacent elements in a uniformly spaced array are first combined in pairs each of which forms a null in a particular direction in space. After this combination in an array of N elements, there remain N-1 antenna signals to further combine to form a beam in some other direction. The final array pattern consists of the formed beam with the null superposed. The process is easy to envision for a single frequency and uniformly spaced elements along a straight line. The following discussion generalizes the idea to a non-uniform array and a broadband interfering signal.

We assume that all antennas receive the same signal from any direction in space except for differences in delay. At the reference antenna, the signal is v(t).

$$v(t) = \int_0^\infty a(\nu) cos(2\pi\nu t + \phi(\nu))\, d\nu \tag{1}$$

Signals arriving at other antennas are the same except for the time delays associated with their distance from the reference antenna τ_{ig}. $v_i(t)$=$v(t-\tau_{ig})$. The geometric delay is given by the following.

$$\tau_{ig} = -\vec{S_i} \cdot \hat{n}(t)/c \tag{2}$$

$\hat{n}(t)$ is the instantaneous direction of arrival of the signal, and $\vec{S_i}$ is the location of the ith antenna with respect to the reference.

A block diagram appears below. Two of the antennas are shown. Following the antennas are mixers to baseband. These include the addition of phase. In fact, the phase is more accurately added at baseband after the signal has been digitized. Needed delay is also added in the baseband digital stream. Exactly how the phase and delay are added does not affect the argument. As shown, the output of each antenna is subtracted from that of its neighbor after a suitable delay has been added to it.

The subtraction of $\pi 2/$ converts the cosine of the argument in (1) to sine. With the inclusion of the mixing with the oscillator at ν=ν_0 and the addition of phase ϕ_{0i}, the output voltage (1) becomes

$$v_i' = -\int_0^\infty a(\nu) sin[2\pi(\nu-\nu_0)(t-\tau_{ig}) - 2\pi\nu_0\tau_{ig} - \phi_{0i} + \phi(\nu)]\, d\nu \tag{3}$$

As shown in the previous memo, two of the terms in the argument of (3) cancel each other by the fringe rotation.

$$-2\pi\nu_0\tau_{ig} - \phi_{0i} = 0 \tag{4}$$

The indicated subtraction of the two signals leaves

$$v_{i+1}''(t) = -\int_0^\infty a(\nu)(sin[2\pi(\nu-\nu_0)(t-\tau_{(1+i)g}) + \phi(\nu)]\, d\nu \tag{5}$$

$$-sin[2\pi(\nu-\nu_0)(t-\tau_{(i)g}-\tau_{in})+\phi(\nu)])\,d\nu \tag{6}$$

The appropriate trigonometric identity is:

$$sinA-sinB=2sin[(A-B)/2]cos[(A+B)/2] \tag{7}$$

For the nulling delay, we choose:

$$\tau_{in}=-\hat{n}_n\cdot(\vec{S}_{i+1}-\vec{S}_i)/c \tag{8}$$

The final result for $v''_{i+1}(\mathrm{t})$ is:

$$\int_0^\infty a(\nu)sin\pi(\nu-\nu_0)[\vec{S}_{i+1}-\vec{S}_i]/c\cdot(\hat{n}_n-\hat{n})\times cos[2\pi(\nu-\nu_0)(t-(\tau_i+\tau_{i+1}+\tau_{in})/2)+\phi(\nu)]\,d\nu \tag{9}$$

A main beam can be synthesized from the sum of all the signals at the next layer, that is, by adding up all the voltages $v''_{1+i}(\mathrm{t})$ with the proper delays, as discussed in the previous memo. The number of available voltages is now N-1, a loss of only one antenna.

3 Discussion

When $\hat{n}=\hat{n}_n$, the argument of the sine function, the first factor, is zero, and $v''_{i+1}(\mathrm{t})$ is zero. That is, the array has a null response for the entire voltage corresponding to a source in that direction. This is a broadband null. In other directions, the voltage response is not zero. Indeed, (9) is similar to the input voltage $v_{i+1}(\mathrm{t})$ except for the small added delay, $\tau_{in}/2$, and, more importantly, the sin( ) factor which weights $a(\nu)$. The latter factor is somewhat frequent dependent, and it is different for each antenna, because the nulling pairs will have slightly different separations and orientation. If the array is laid out as a long string of approximately equally separated antennas, a good choice for the cable hook-up, then the sin( ) factors will be very similar for all the terms. If the typical adjacent separations are too large, then there will be multiple nulls associated with each pair and more likelihood of one of them falling in the direction of the main beam. One or more of the null pairs may put a null on the main beam. In that case, the subtraction of the pair producing that null should not be made. The slow frequency dependence will mean that the signal that is detected in the main beam will be only slightly distorted. However, the sin( ) factor will reduce the sensitivity in the main beam and lower the signal/noise ratio in the main beam. On the average, this effect should not exceed a factor 2.

Although this scheme may be workable, it may be inferior to the interference subtraction scheme outlined in the previous memo. In that method, the desired signal appears to suffer less distortion, unlike the case here. However, the other scheme does result in some distortion, and a more detailed comparison is needed to determine which may be better. One advantage of the present scheme is that it does not appear to be as sensitive to the interference/noise level as does the other scheme.

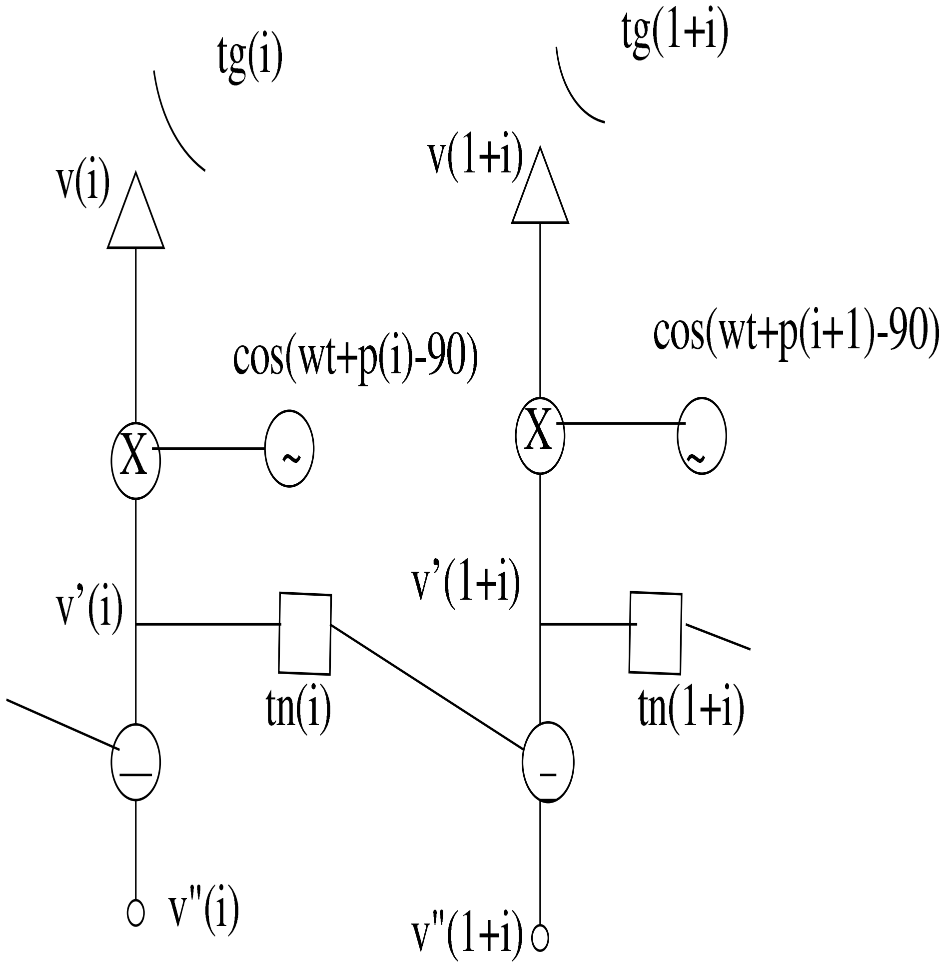

Figure 1: Block diagram showing two of the antennas with the interconnection which produces a null. The geometric delays are shown at the top next to the antennas. The row of outputs with voltages labeled v" can now be used to form the main beam in some other direction. There is one less antenna voltage to form this beam as a result of the nulls put into the top row. In the figure, Roman symbols are used instead the Greek in the text. For example, tn is used forτ_n.